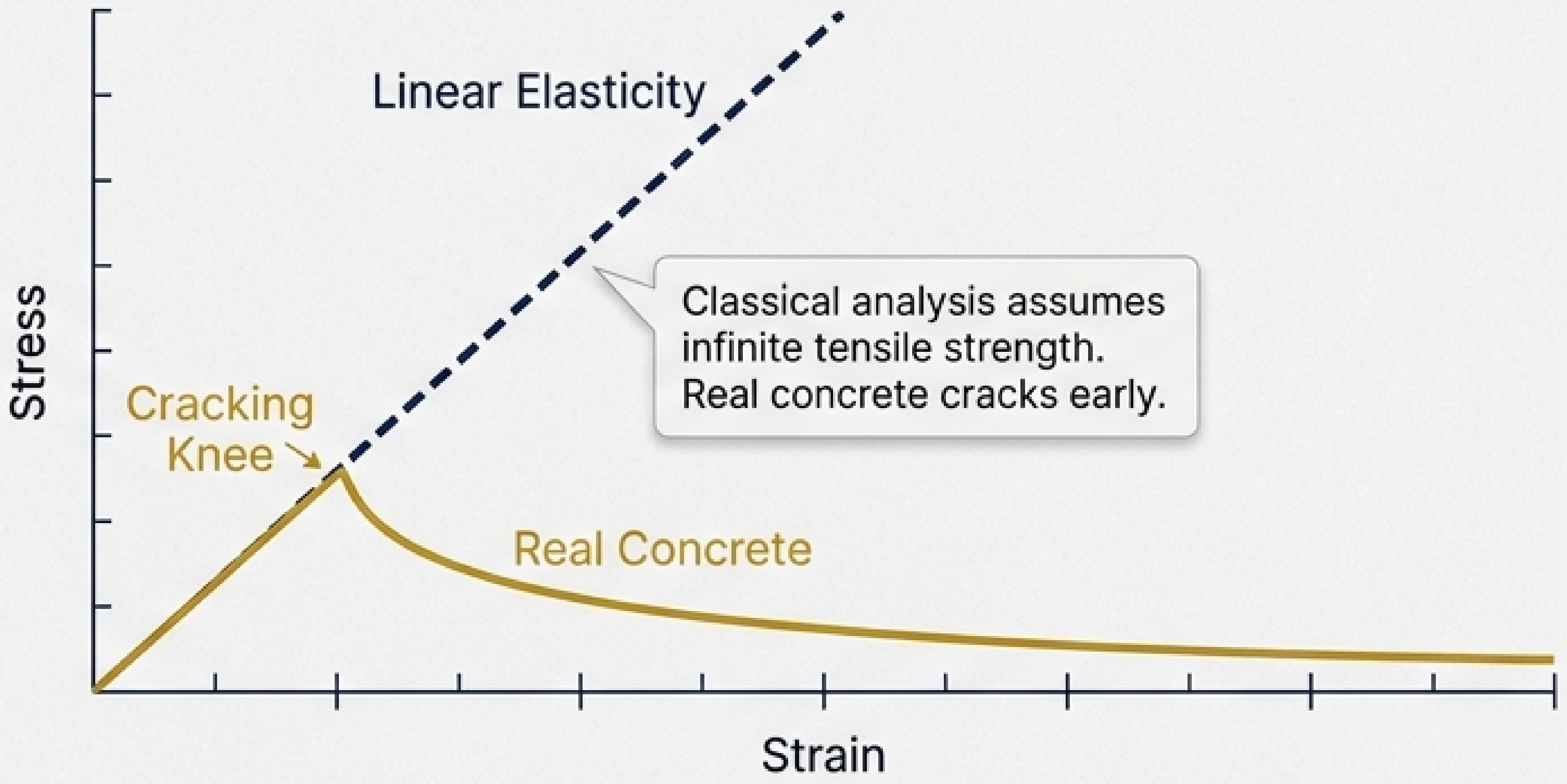
ADVANCED NUMERICAL ANALYSIS

Beyond Linear Limits: Simulating Structural Reality with ATENA.

A Fortune 500 technical dossier on nonlinear finite element analysis, fracture-plasticity theory, and risk mitigation in reinforced concrete.

Červenka Consulting · Structural Engineering Masterclass.

Classical linear analysis fails exactly where concrete begins to fail.



Reinforced concrete is strongly nonlinear. It cracks in tension, crushes in compression, and relies on embedded steel to bridge gaps. Linear analysis cannot capture behavior after cracking initiates.

The Nonlinear Solution

$$K(d) \cdot \Delta d = F - R(d)$$

Solves equilibrium incrementally, tracing the true structural response—including serviceability, crack widths, and ultimate ductility.

THE ANALYTICAL DIVIDE

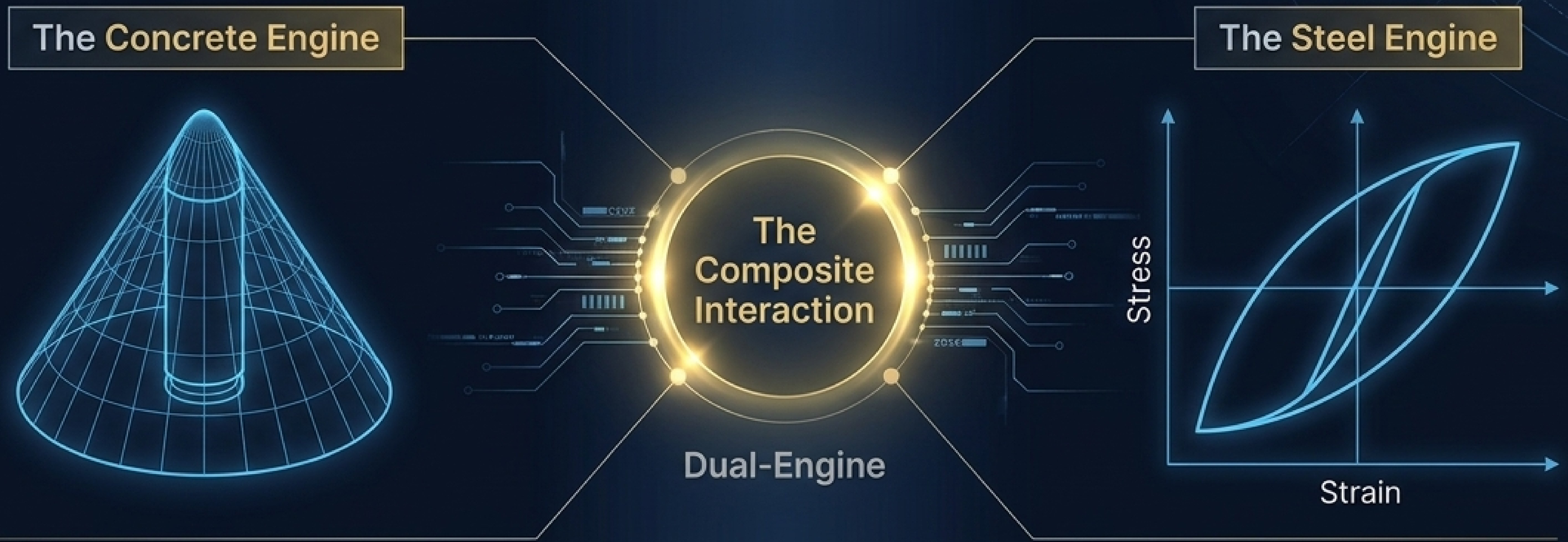
Linear vs. Nonlinear FEA: The shift from assumption to simulation.

	Linear Analysis	Nonlinear FEA (ATENA)
Handling of Cracking	Ignored (Assumes infinite elasticity)	Simulated via **smeared-crack & fracture mechanics**
Serviceability Prediction	Calculated via empirical hand-formulas	Directly computed from **bond-slip & crack widths**
Failure Mode Prediction	Cannot predict shear vs. flexure limits	Reveals **true failure mode** (e.g., diagonal shear cracks)

For sheer capacity and risk mitigation, capturing the post-peak softening branch is non-negotiable.

THE CONCEPTUAL FRAMEWORK

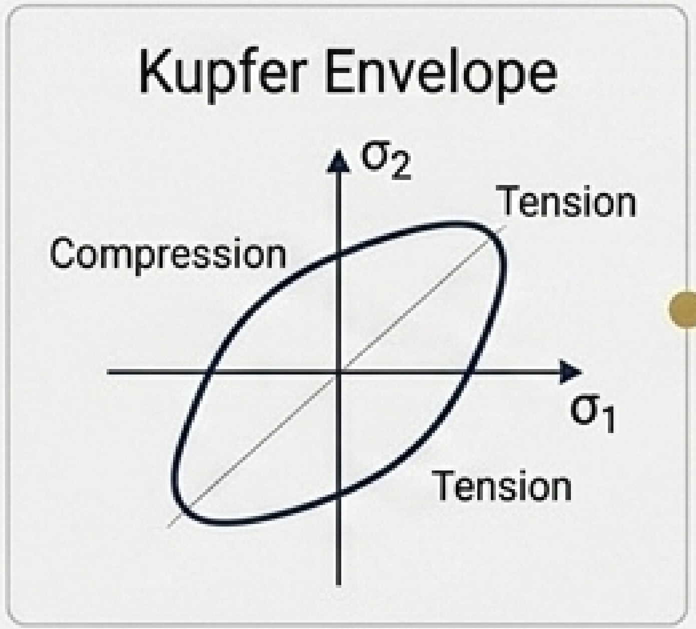
The twin theoretical pillars of composite structural simulation.

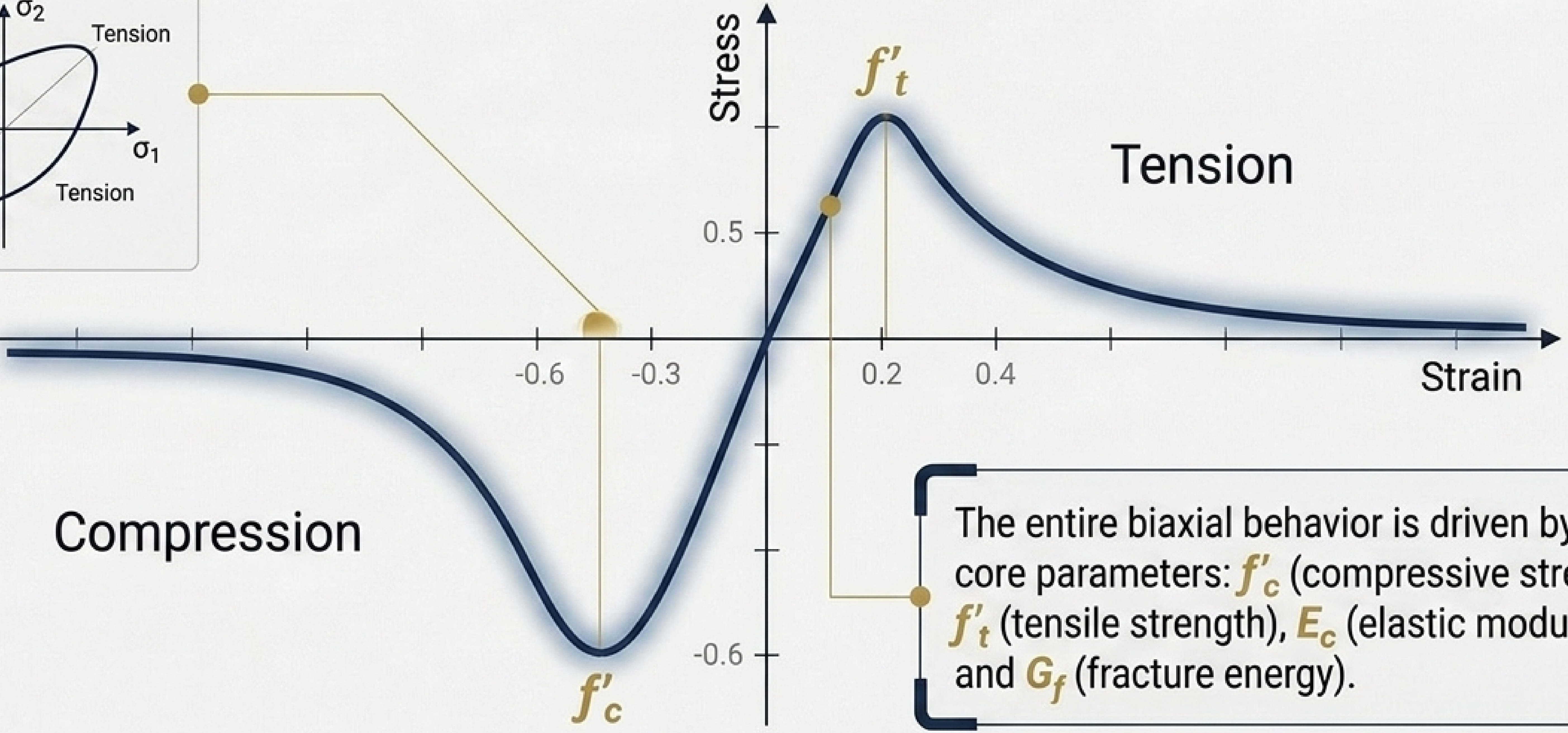


- Rankine Tensile Fracture
- Menétrey-Willam Plasticity

- Yield Hardening
- Bond-Slip Integration

Fracture-plasticity: A unified law for cracking and crushing.



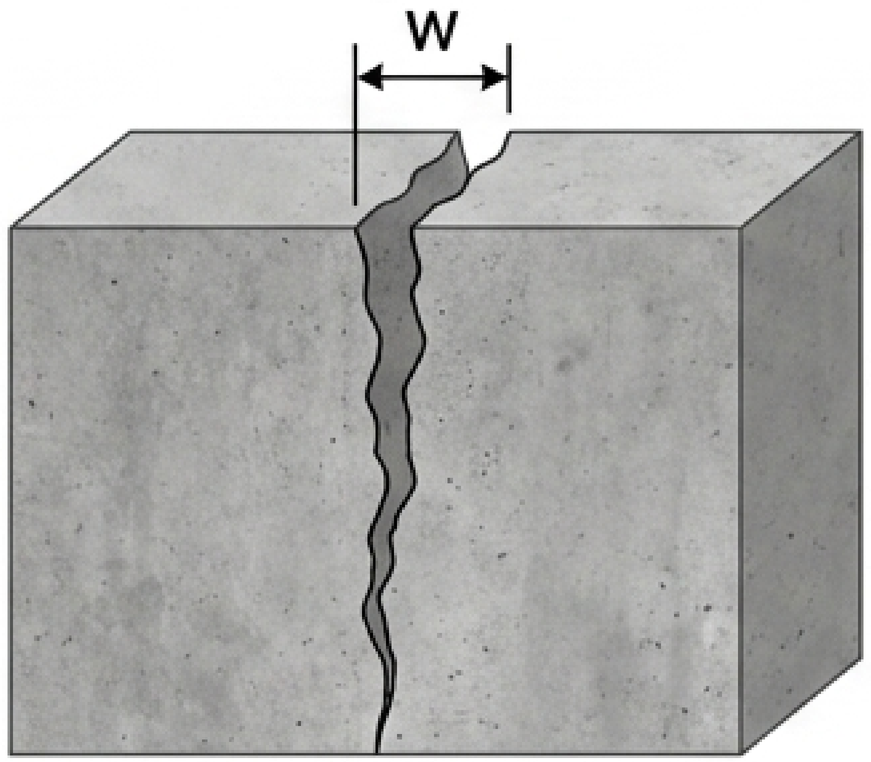


The entire biaxial behavior is driven by four core parameters: f'_c (compressive strength), f'_t (tensile strength), E_c (elastic modulus), and G_f (fracture energy).

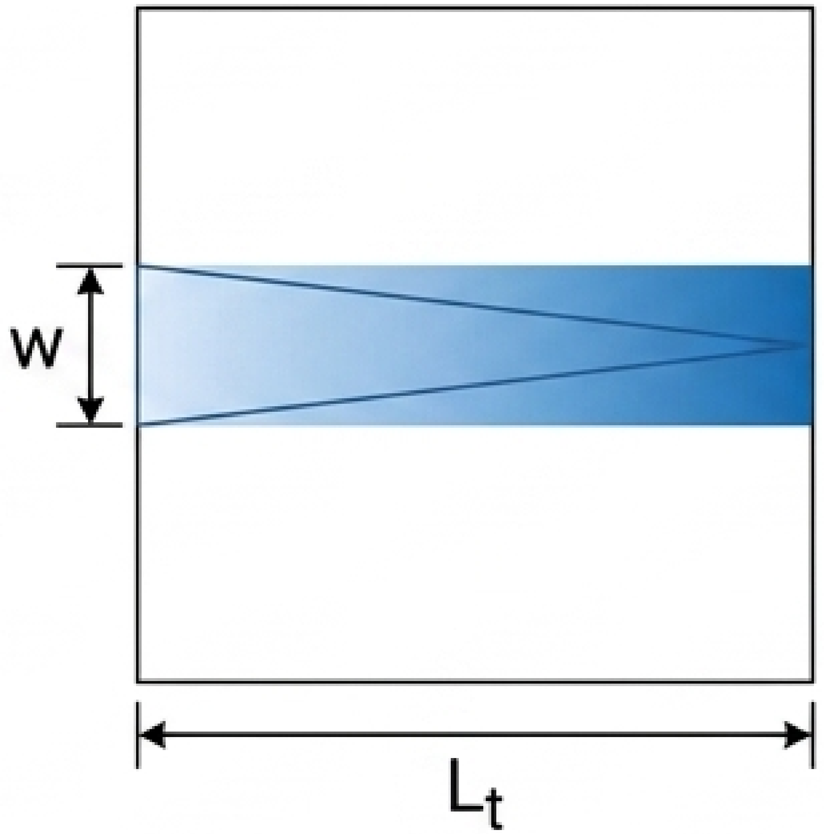
NUMERICAL ROBUSTNESS

The Mesh-Objectivity Mechanism: Regularizing the crack band

Physical Crack



Smeared Element



Regularization

$$\varepsilon_{cr} = \frac{w}{L_t}$$

ATENA applies Bažant and Oh’s crack-band model. By smearing the crack opening (w) over the element length (L_t), the dissipated energy equals the physical fracture energy (G_f)—regardless of mesh density.

Without regularization, refining the mesh artificially reduces computed structural strength.

Formulating the fracture: Fixed vs. Rotated crack models.

Fixed Crack Model

Concept: Crack direction locks at the moment of initiation.

Shear: Subsequent shear is carried through a variable shear-retention factor.

Advantage: Highly physically accurate for localized cracking.

Rotated Crack Model

Concept: Crack direction co-rotates to stay normal to current principal strain.

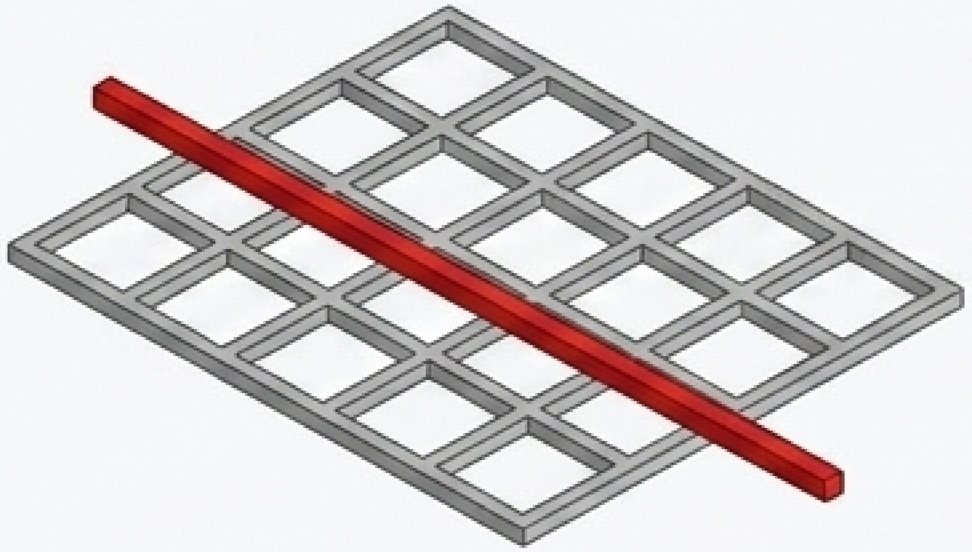
Shear: No shear stress develops on the crack plane.

Advantage: Numerically robust; excellent for generalized damage.

ATENA allows users to choose, or even blend, both formulations to balance physical accuracy with mathematical convergence.

Reinforcement architectures: Discrete bars vs. Smeared layers.

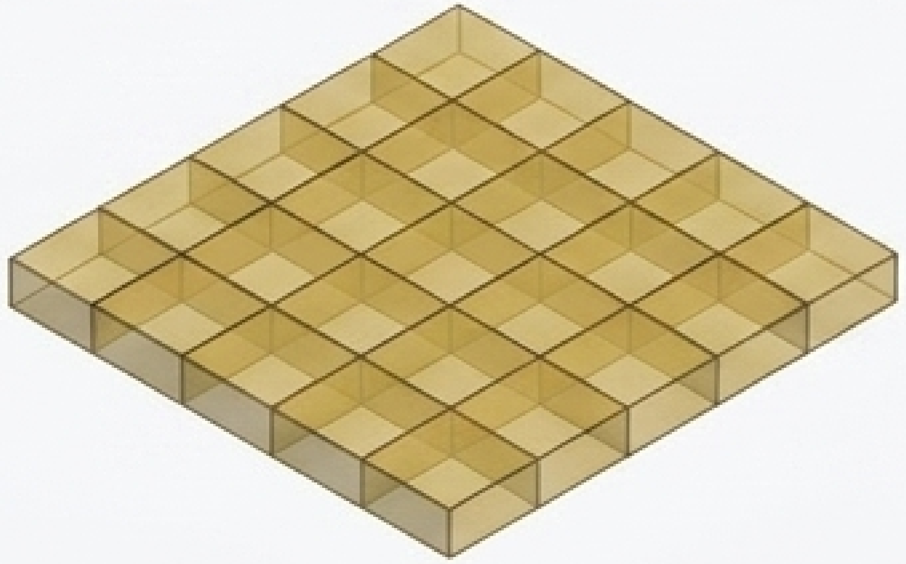
Discrete Reinforcement



Formulation: Trusses embedded in the continuum.

Use Case: Ideal for beams where exact bar and stirrup location governs failure.

Smeared Reinforcement



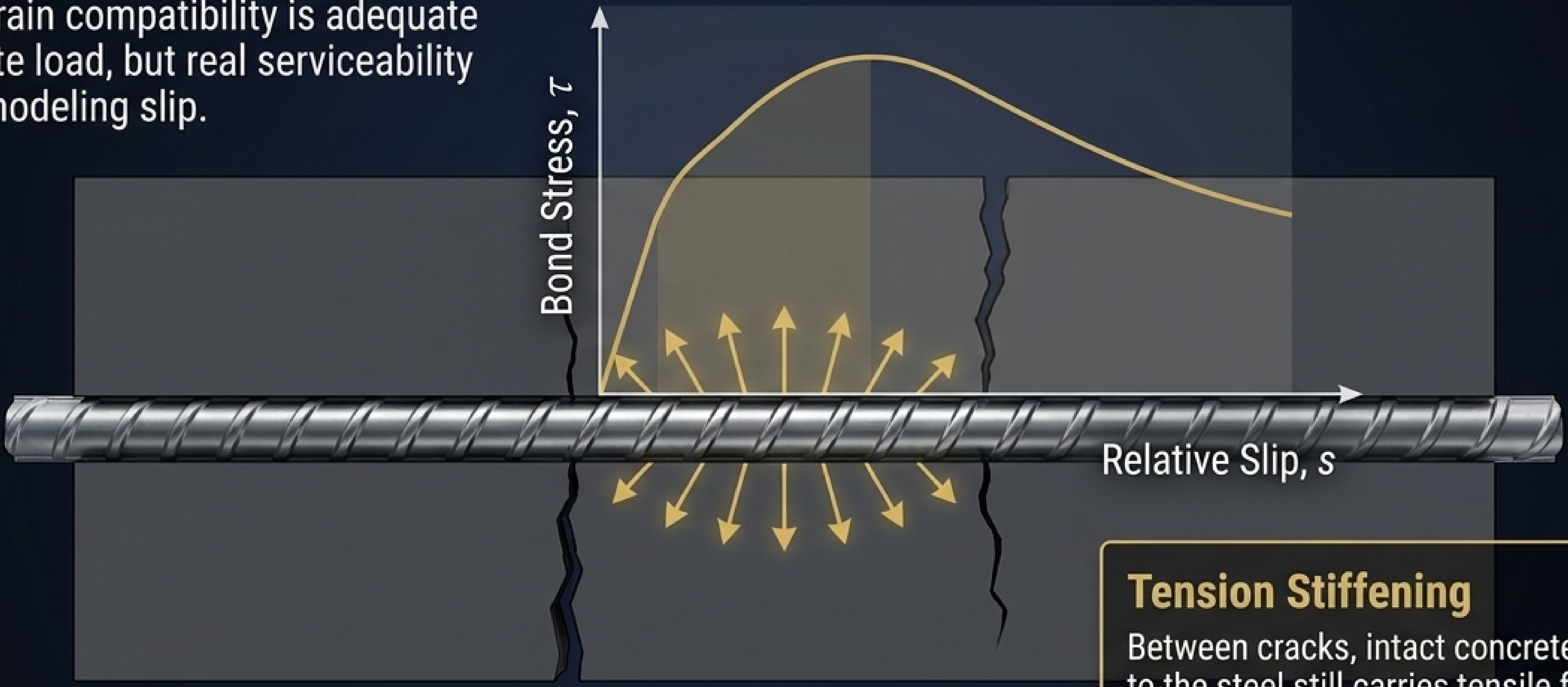
Formulation: $\sigma_s = \sum p \cdot T \cdot \sigma'_s$

Use Case: Ideal for slabs or shells with dense, regular meshes.

SYNTHESIS: THE COMPOSITE INTERACTION

Bond-slip transfer: Concrete is never mathematically dead.

Perfect strain compatibility is adequate for ultimate load, but real serviceability requires modeling slip.



Tension Stiffening

Between cracks, intact concrete bonded to the steel still carries tensile force. ATENA models this force transfer, ensuring accurate crack spacing and realistic deflection predictions.

IMPLEMENTATION & WORKFLOW

From blueprint to reality: The 6-step ATENA timeline.

1. Materials
SBETA Concrete & Bilinear Steel.

2. Geometry
Joints → Lines → Macro-elements.

3. Reinforcement
Discrete bar pathing.

4. Boundary Limits
Symmetry & Supports.

5. Nonlinear Solution
Displacement-controlled iteration.

6. Post-Processing
Cracks, strains, and capacity.

Applied to the classic Leonhardt shear beam—a geometry designed to test the limits of diagonal shear failure.

STEP 1 & 2: GEOMETRY

Bottom-up digital twin architecture

1

Data-Access Tree

Joints
↳ Lines
↳ Macro-Elements

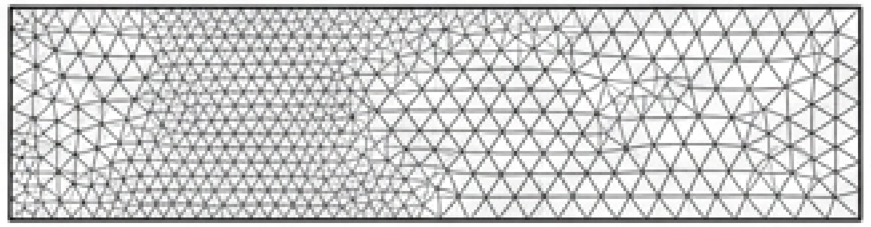
2

Material Generation

Cube Strength	33.5 MPa
f'_c	-28.5 MPa
f'_t	2.4 MPa
G_f	5.6e-5 MN/m

3

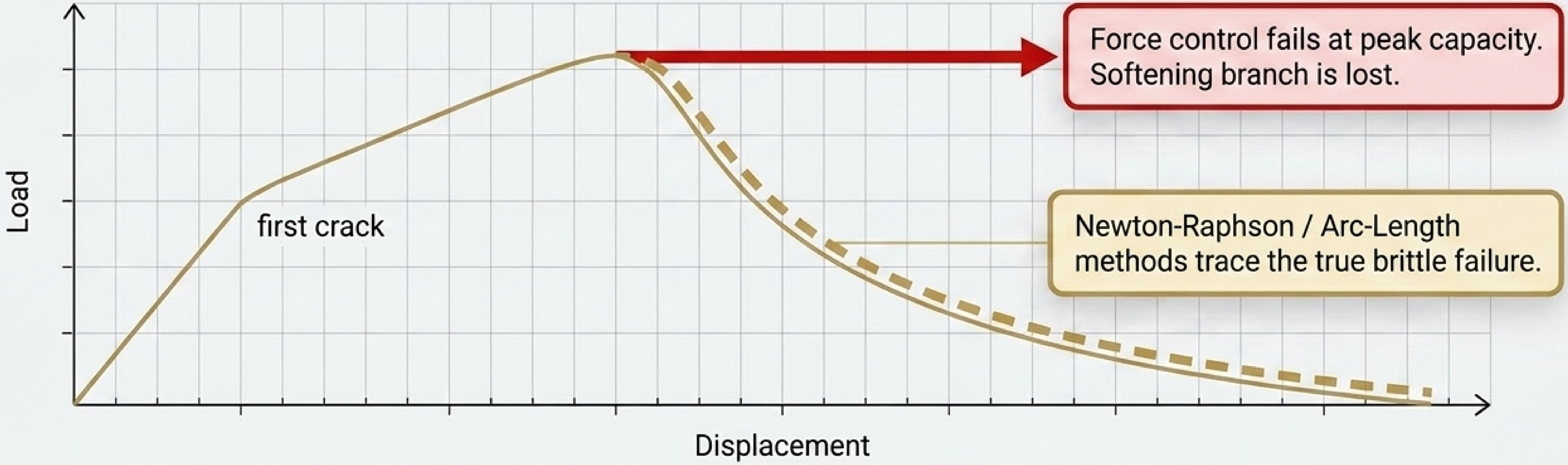
Automated Meshing



ATENA 2D builds topology hierarchically. The automatic mesh generator fills macro-elements with low-order elements, optimized for the crack-band regularization theory.

STEP 4 & 5: THE SOLUTION STATE

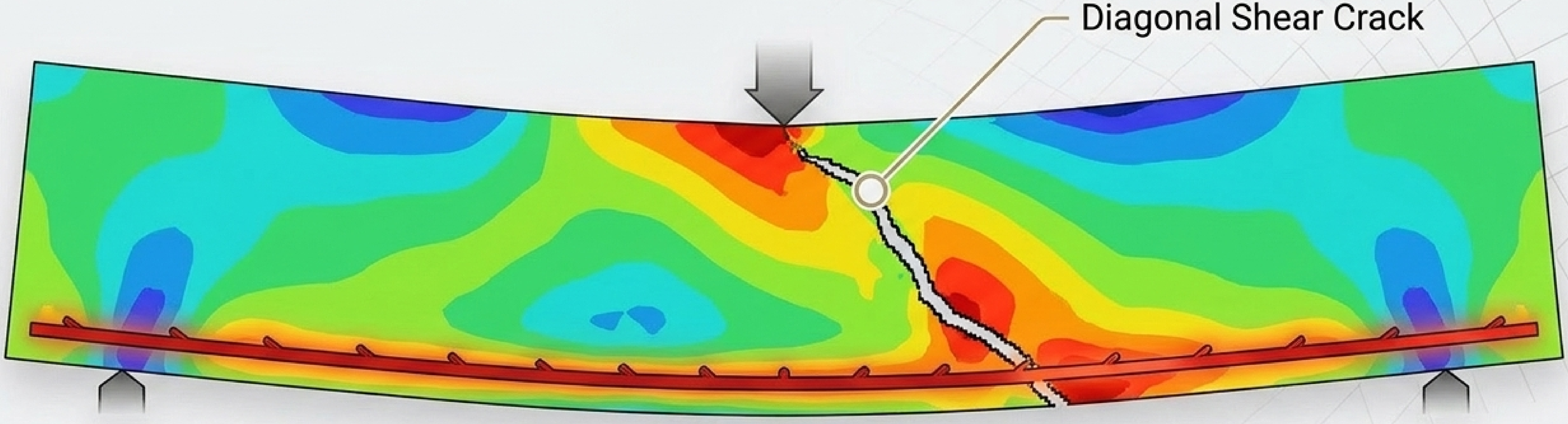
Capturing the snap-back: The necessity of displacement control.



To evaluate structural ductility, the solver must follow the structure past peak load. ATENA achieves this by incrementally enforcing displacement, not load.

STEP 6: POST-PROCESSING

Revealing the hidden failure mechanism.



Linear analysis predicts bending. ATENA predicts reality. The principal-strain plot reveals an abrupt, limited-ductility diagonal shear crack—the exact mechanism governing this specific geometry.

ASSURANCE & RELIABILITY

An executive framework for model validation.

1. Mesh-Sensitivity Check

Do finer meshes yield the same peak load? Validates crack-band effect.

2. Parameter Audit

Are f'_t and G_f calibrated correctly? They drive brittleness.

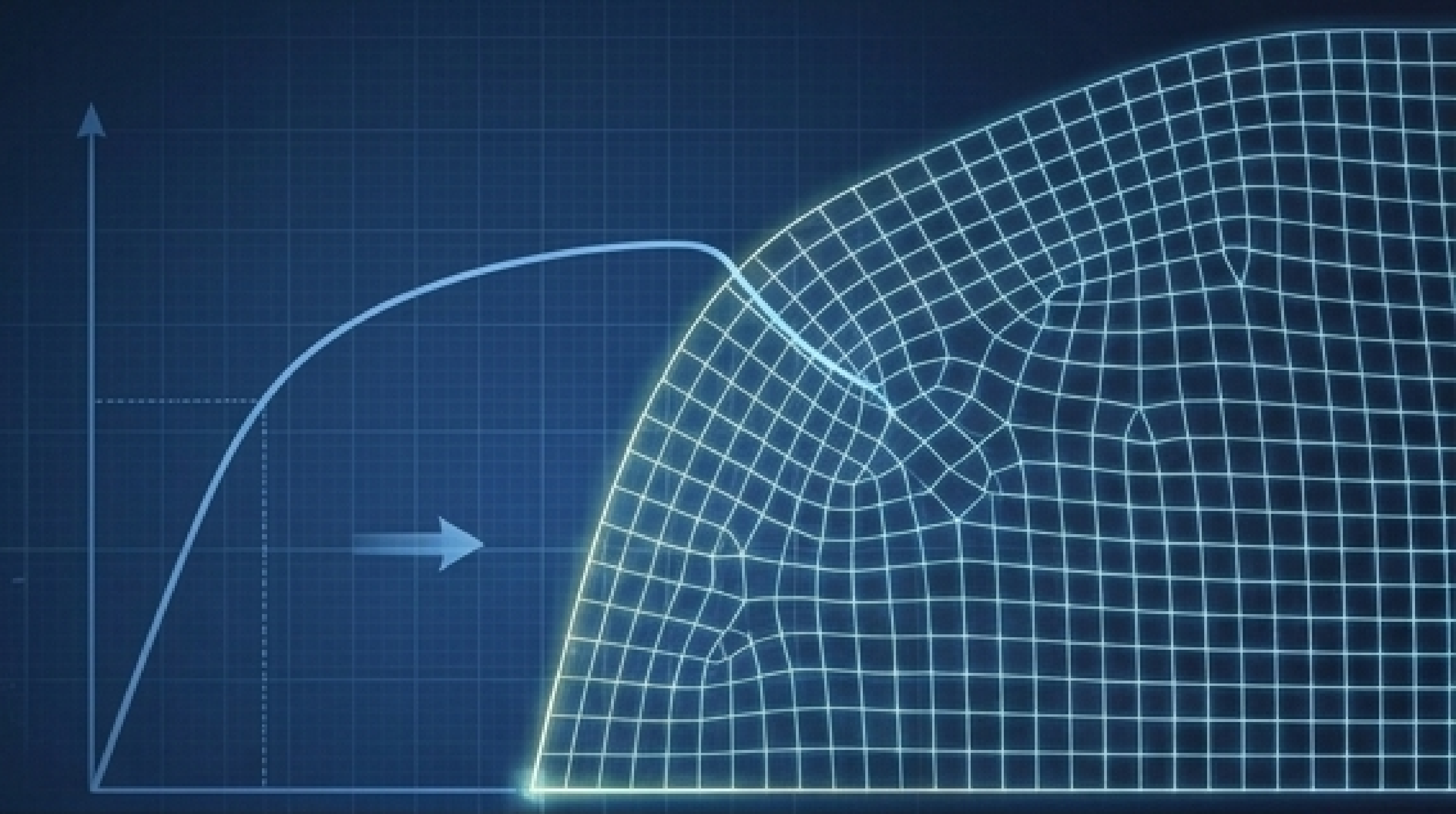
3. Experimental Benchmark

Does the load-displacement curve match physical lab tests?

Common Sources of Error: Force instead of displacement control, coarse meshing in high-gradient zones, and incorrect symmetry boundary conditions.

THE ATENA STANDARD

Redefining structural reliability through computable reality.



1. Theoretical Rigor	2. Composite Fidelity	3. Uncovered Risk
Mesh-objective predictions through coupled fracture-plasticity and crack-band regularization.	Uncompromising accuracy via discrete reinforcement, bond-slip, and tension stiffening.	Exposing brittle shear failures and post-peak softening that classical methods obscure.

If the load-displacement curve, the crack pattern, and the failure mode agree with physical expectation, the model is trustworthy. **ATENA** makes this possible.